

Minimax: connecting game theory and social justice

Michael Price

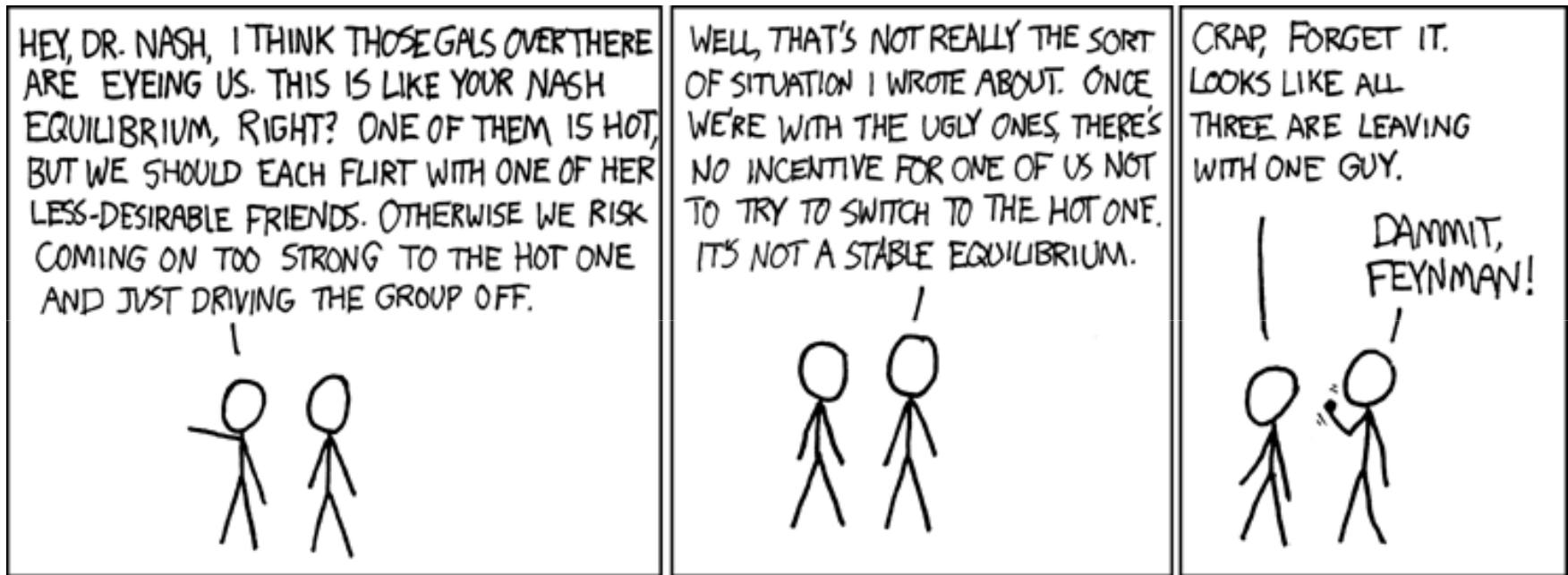
mhprice@stanford.edu

www.michaelholtonprice.com

Roadmap

1. Audience background survey
 - Game theory
 - Math
 - Social justice
 - Poker
- What is game theory?
- Game theory vs. optimization

xkcd introduces game theory



What is game theory?

- I really like this definition: “Game theory is the branch of mathematics that deals with decision-making in situations in which two or more players have competing interests.”
 - <http://www.cardplayer.com/magazine/article/14980>
- Pioneered by the uber-geek John von Neumann circa 1930's

Applications

- Widely used in the social, life, and physical sciences (and elsewhere!)
- Parliamentary voting blocks
- Oligopolies
- Thermonuclear warfare (first strike)
- Biological evolution
- Cultural evolution (my field)
- Poker (my other field)

Let's get our hands dirty...

- Who are the competing agents in the xkcd comic?
 - Male 0 and Male 1
 - [A more sophisticated model might account for the interests of the females and Feynman]
- What (er, who) are they competing over?
 - Hottie and other females
- What are the males' strategy alternatives?
 - Each male may hit on the hottie or the others (potentially make the same choice... no chickening out and hitting on no female(s))

“Normal form” (matrix) representation of xkcd game

	Hottie	Others
Hottie	1 / 1	10 / 0
Others	0 / 10	0 / 0

There's no incentive to be altruistic!

Different characteristics of games

- Two person
- Zero sum
- Perfect/imperfect recall
- Perfect/imperfect information
(check/poker)
- Typically games are represented in normal
(matrix) form or extensive (tree) form
- Pure versus mixed strategies (next slide)

Prisoner's dilemma to illustrate pure versus mixed strategies

	Cooperate	Defect
Cooperate	3 / 3	0 / 5
Defect	5 / 0	1 / 1

Pure strategy: Always cooperate

Mixed strategy: Cooperate 70%, Defect 30 %

Nash equilibrium and the minimax theorem

- At a Nash equilibrium no player can gain by unilaterally deviating from their (possibly mixed) equilibrium strategy
- For a finite, two person, zero sum game minimizing your opponents maximum gain yields a Nash equilibrium
 - This is the minimax theorem
 - Since the game is zero sum, you also maximize your minimum gain (this is not intuitively obvious to me)
 - All the proofs of the minimax theorem I've seen utilize the fact that the any mixed strategy response to a given opponent's strategy is at least as good as the worst pure strategy response

Game theory is not optimization

- If your opponent is dumb enough to publish his/her strategy (and stick to it) game theory is inapplicable
- Instead, you solve an optimization problem to maximize your gain given your opponent's published (possibly mixed) strategy

Rock-paper-scissors

- Rock paper scissors is a finite, two person, zero sum game
- The only equilibrium is a mixed strategy in which one chooses each option with probability $1/3$ (game theory)
- If, however, your opponent publishes a strategy of scissors you should play rock (optimization)
 - Aside: it makes sense that rock beats scissors and scissors beats paper, but why should paper beat rock? Oooh, paper covers rock. So scary.
Pedagogically, I prefer rock-dynamite-scissors

A digression into maximin in Rawls' *A Theory of Justice*

- Note: this is a digression into maximin as a decision rule (optimization). It illustrates the danger of applying game theoretic ideas to optimization, and vice versa
- In a Theory of Justice Rawls describes the original position, a hypothetical device useful for examining the properties of a fair social contract among rational individuals. In the original position the rational individuals are constrained by a veil of ignorance
 - *"[N]o one know his place in society, his class position or social status; nor does he know his fortune in the distribution of natural assets and abilities, his intelligence and strength and the like... More than this, I assume that the parties do not know the particular circumstances of their own society. That is, they do not know its economic or political situation, or the level of civilization and culture it has been able to achieve."*

maximin in the veil of ignorance

- Under the veil of ignorance, a group of rational individuals choose a social contract they deem “fair.” Note that Rawls’ problem is one of optimization, not game theory
- Rawls examines a number of definitions of fair (such as maximizing total utility), in the end preferring maximin.
- Huh?
- Here’s his reasoning. An unequal distribution of goods (capitalism) is unfair unless it improves the lot of the least happy person in society
- That’s very compelling, but...

maximin has some serious failing as a decision rule

- How do you deal with sadists? Or pedantic bores and primadonnas, who need attention in order to be happy?
- If a pedantic bore is the least happy person in society, should he (uh, or she) be allowed to lecture happier people, thus significantly reducing total unhappiness?
- Harsanyi argues convincingly that the answer is no
 - John C. Harsanyi, “Can the Maximin Principle Serve as a Basis for Morality? A Critique of John Rawls’ Theory,” *American Political Science Review* 69 (1975): 594-606.
- But for a rejoinder see
 - <http://homepage.uab.edu/angner/pdf/Rawls.pdf>

I had an interesting idea recently, though

- Let's say a politician has money to spend on advertizing (or pander to or bribe)
- Who should the politician target?
- Well, if you give a rich person \$10,000 it won't make them much happier, but if you give a poor person \$10,000 they'll be ecstatic
- So, the greatest bang for the political buck comes from pandering to the least advantaged segment of the electorate (maximin!!)