Bayesian Reconstruction of Past Demography

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Society for American Archaeology
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Follow Along / Run the Code Yourself

http://www.michaelholtonprice.com/saa2018

yada: Yet Another Demographic Analysis package

http://www.michaelholtonprice.com/yada

Relevant Data

Direct

Relevant Data

Direct

- Date of Death [C14]

Relevant Data

Direct

- Date of Death [C14]
- Age at Death

Relevant Data

Direct

- Date of Death [C14]
- Age at Death
- Isotopes
- DNA

Relevant Data

Direct

- Date of Death [C14]
- Age at Death
- Isotopes
- DNA

Indirect

- Pottery
- Charcoal
- House Counts
- etc.

Approach: Fuse Data in a Fully Bayesian Way

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Status

- Supporting math at end of presentation click to go there

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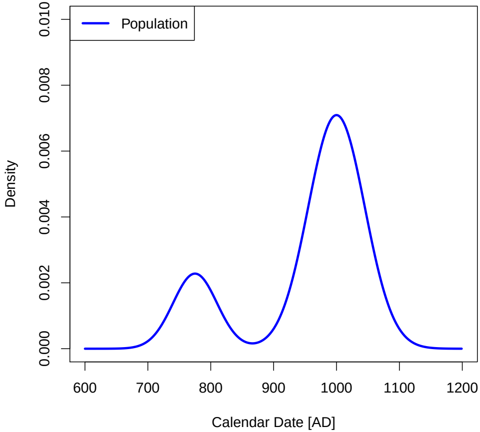
- In work: Age-at-Death see some math

Approach: Fuse Data in a Fully Bayesian Way

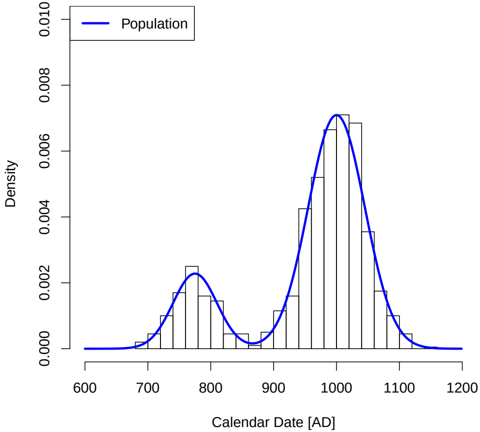
Status

- Supporting math at end of presentation click to go there
- Implemented in yada: Date-of-Death [C14]
- In work: Age-at-Death see some math
- Future: Isotopes, DNA, etc.

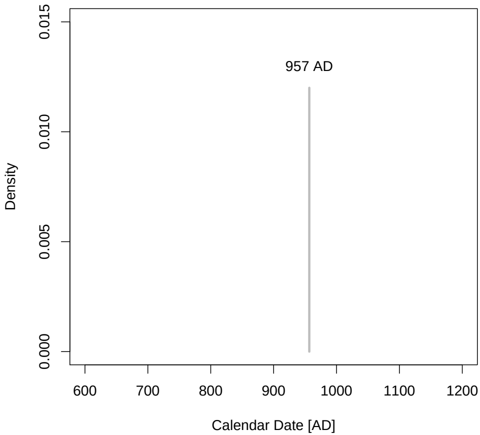
Simulation: Target Curve



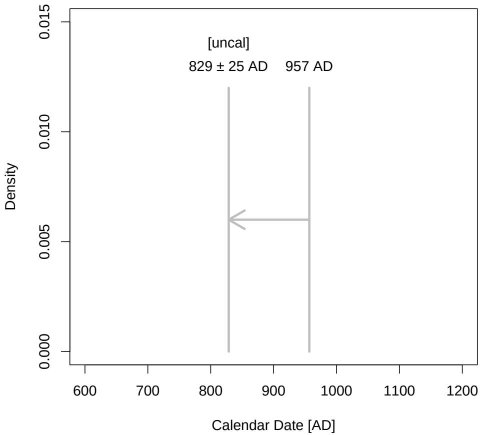
Simulation: 1000 Draws for Date-of-Death



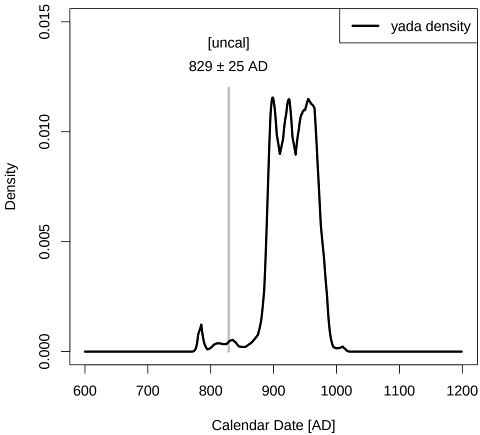
Simulation: Date-of-Death to Radiocarbon Meas.



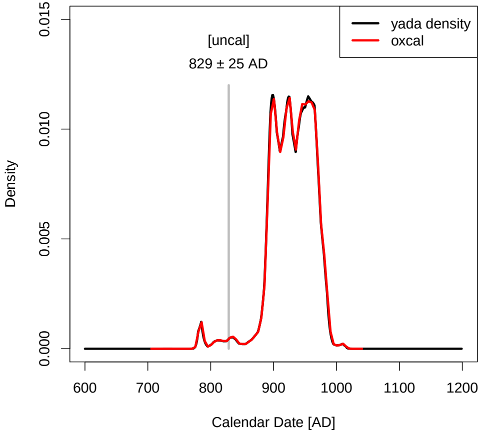
Simulation: Date-of-Death to Radiocarbon Meas.



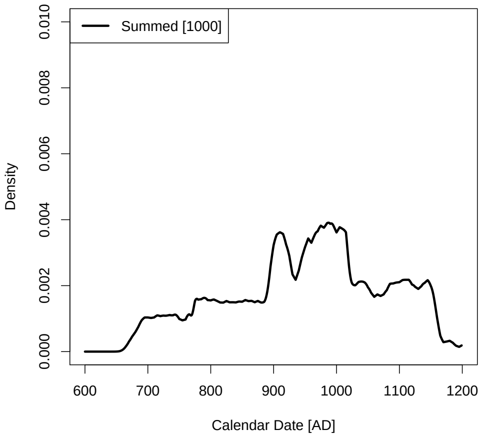
Measurement and Calibration Curve Uncertainty



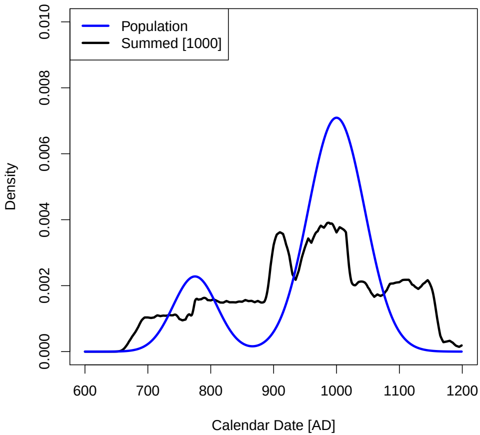
Measurement and Calibration Curve Uncertainty



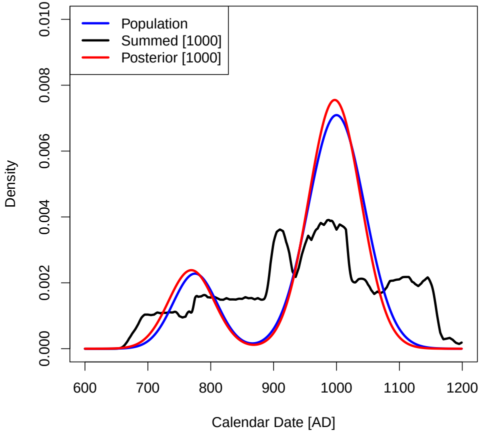
Summed Probability Densities [N=1000]



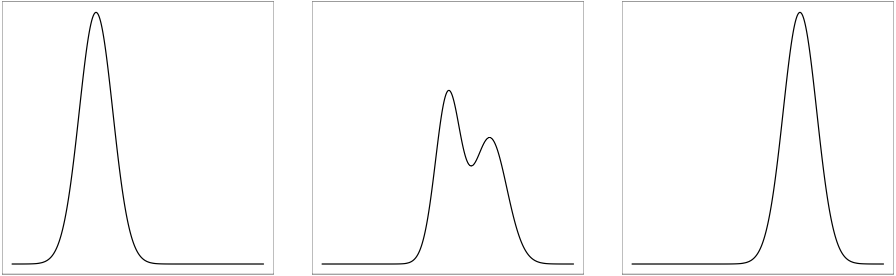
Summed Probability Densities vs. Target Population



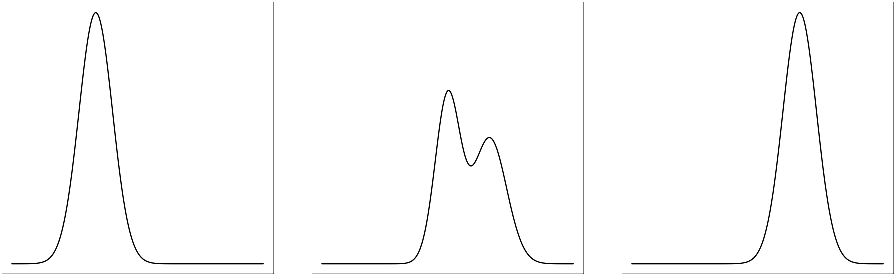
Summed Probability Densities vs Fully Bayesian Model



Bayesian Inference

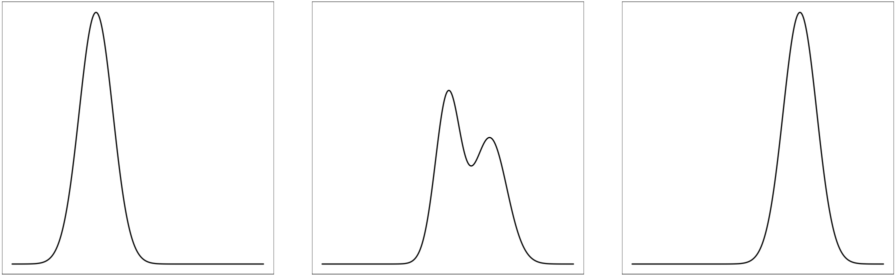


Bayesian Inference



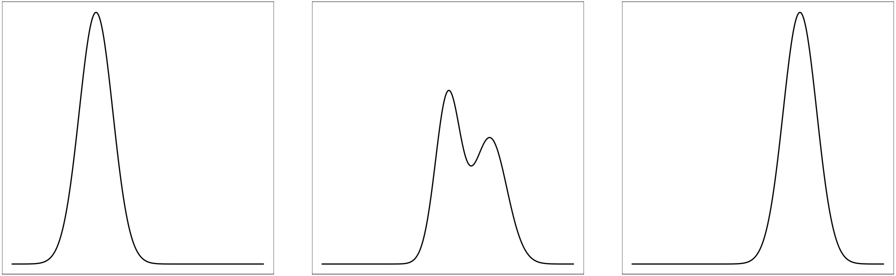
Prior 0.40 0.20 0.4

Bayesian Inference



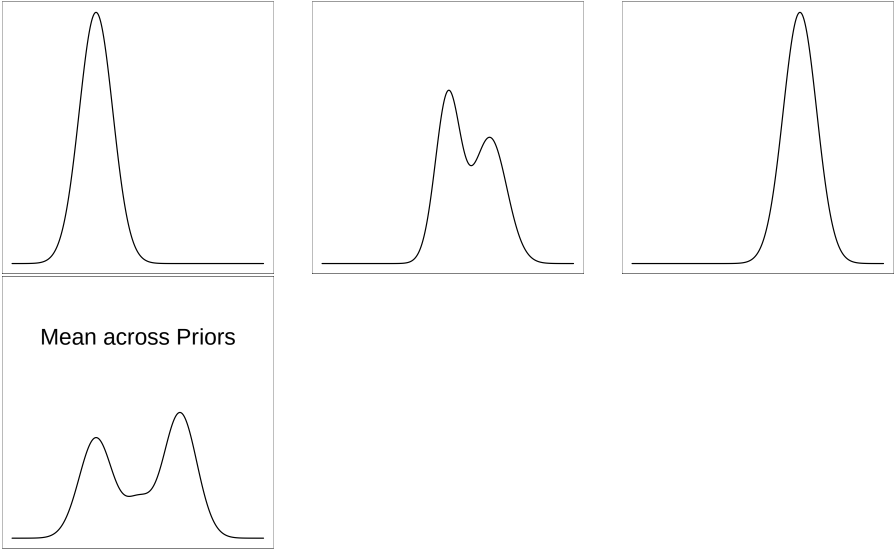
Prior	0.40	0.20	0.4
Likelihood	0.02	1.00	0.04

Bayesian Inference

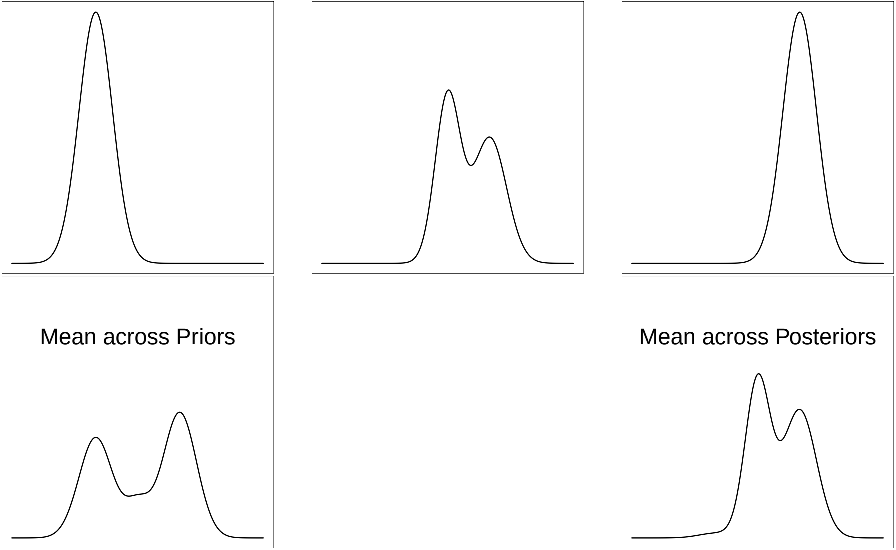


Prior	0.40	0.20	0.4
Likelihood	0.02	1.00	0.04
Posterior	0.02	0.94	0.04

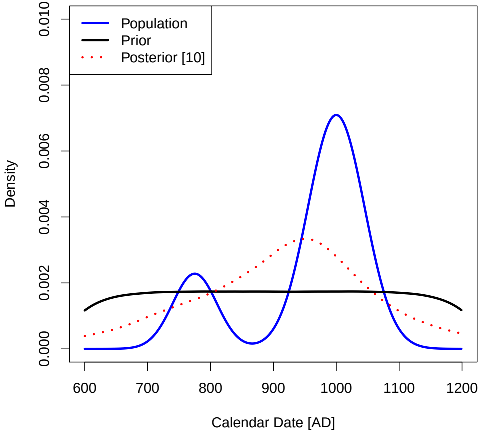
Bayesian Inference



Bayesian Inference

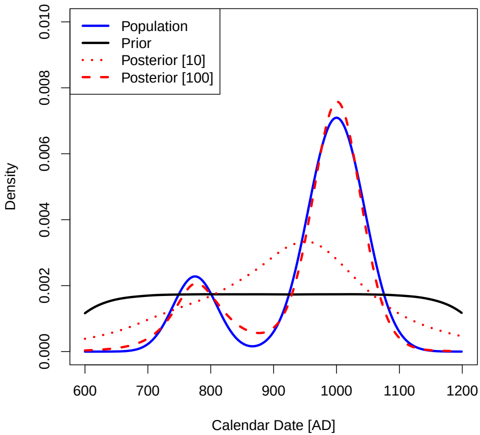


Sample Size



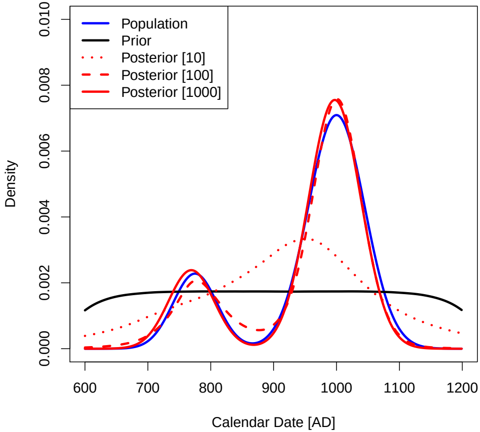
Population
Prior
Posterior [10]
Density
0.000
0.002
0.004
0.006
0.008
0.010
600
700
800
900
1000
1100
1200
Calendar Date [AD]

Sample Size



Population
Prior
Posterior [10]
Posterior [100]
Density
0.000
0.002
0.004
0.006
0.008
0.010
600
700
800
900
1000
1100
1200
Calendar Date [AD]

Sample Size


Population
Prior
Posterior [10]
Posterior [100]
Posterior [1000]
Density
0.000
0.002
0.004
0.006
0.008
0.010
600
700
800
900
1000
1100
1200
Calendar Date [AD]

Acknowledgements

Kyle Bocinsky
Julie Hoggarth
Tim Kohler
Jamie Jones
Kyra Stull
Brendan Tracey
Shripad Tuljapurkar

Supporting Math

$$p(\boldsymbol{\theta}|\mathbf{D},\boldsymbol{\alpha}) = \frac{p(\mathbf{D}|\boldsymbol{\theta})\, p(\boldsymbol{\theta}|\boldsymbol{\alpha})}{p(\mathbf{D}|\boldsymbol{\alpha})}$$

$\boldsymbol{\alpha}$:= Hyperparameter specifying priors

$\boldsymbol{\theta}$:= Parameter for demographic models

$\mathbf{D}$:= Data

y := Calendar year [for later slides]

a := Age-at-death [for later slides]

Likelihood

$$p(\phi_m|\boldsymbol{\theta}) = \int_Y p(\phi_m|y)\, p(y|\boldsymbol{\theta}) dy$$

$$\phi_m|y \sim \mathcal{N}(\mu_\kappa(y), \sqrt{\sigma_\kappa(y)^2 + \sigma_m^2})$$

Integrated Prior/Posterior

$$p(y|\boldsymbol{\alpha}) = \int_{\Theta} p(y|\boldsymbol{\theta})\, p(\boldsymbol{\theta}|\boldsymbol{\alpha}) d\theta$$

$$p(y|\mathbf{D}, \boldsymbol{\alpha}) = \int_{\Theta} p(y|\boldsymbol{\theta})\, p(\boldsymbol{\theta}|\mathbf{D}, \boldsymbol{\alpha}) d\theta$$

Demographic Models: Given Age-at-Death

$$p(y|\boldsymbol{\theta}) \longrightarrow p(y, a|\boldsymbol{\theta})$$

Leslie Matrices

$$
\mathbf{A} = \begin{pmatrix}
F_1 & F_2 & F_3 & F_4 & \cdots & F_M \\
P_1 & 0 & 0 & 0 & \cdots & 0 \\
0 & P_2 & 0 & 0 & \cdots & 0 \\
0 & 0 & P_3 & 0 & \cdots & 0 \\
0 & 0 & 0 & \ddots & \cdots & 0 \\
0 & 0 & 0 & \ddots & P_{M-1} & 0
\end{pmatrix}
$$

Leslie Matrices

Time-independent Leslie matrix:

$$\mathbf{z}_{n+1} = \mathbf{A}\,\mathbf{z}_n$$

$$\mathbf{z}_n = \mathbf{A}\cdots\mathbf{A}\,\mathbf{A}\,\mathbf{z}_0 = \mathbf{A}^t\,\mathbf{z}_0$$

Time-dependent Leslie matrix:

$$\mathbf{z}_n = \mathbf{A}_n\cdots\mathbf{A}_2\,\mathbf{A}_1\,\mathbf{z}_0 = {}_1^n\mathbf{Y}\,\mathbf{z}_0$$

$${}_1^n\mathbf{Y}(\boldsymbol{\theta}) \longrightarrow p(y, a|\boldsymbol{\theta})$$